THE FORMATION OF RAIN BY COALESCENCE

By E. G. Bowen*

[*Manuscript received March* 2, 1950]

Summary

It is generally acknowledged that drizzle or light rain can fall from clouds which do not reach freezing level and cases have recently been described in which moderate to heavy rain has been observed to fall from such clouds. A simple theory is developed to account for the phenomenon, based on the initial growth of cloud droplets by condensation followed by the growth of a small fraction of their number by coalescence. These grow in their ascent through the cloud until they are large enough to remain in suspension in the upward air current, after which they fall as rain. It is shown that for a given set of cloud conditions the maximum height reached by the drops increases with increasing vertical air velocity and that the size of the drops emerging from the base of the cloud is nearly a linear function of the height attained. The time for the precipitation to appear, on the other hand, is an inverse function of the upward air velocity.

Experimental observations of rain from non-freezing clouds have distinguished two main types. The first of these shows an increase in drop diameter or rainfall intensity downward through the cloud, as would be expected if the drops followed a variety of trajectories within the cloud. The second type is one in which the drop trajectories tend to coincide, in which case there would be a maximum in the raindrop density and the rain water content at some defined height within the cloud. This has been verified qualitatively by radar observations and flight experiments.

I. INTRODUCTION

In 1933 Bergeron(1) postulated a mechanism of rain formation in which raindrops began their life as ice crystals and then grew rapidly at the expense of surrounding water droplets. It is now generally accepted that this process plays an important part in the formation of rain from clouds which extend well above the freezing level. At the same time it is recognized that other mechanisms are possible and that drizzle or light rain can fall from clouds which consist wholly of water droplets. In considering this phenomenon Findeisen(2) showed that drops of the required size were not likely to form by condensation alone but could form if coalescence of cloud droplets was taking place. He calculated the rate of growth by coalescence and showed that drizzle would be expected to form in clouds of moderate thickness and large raindrops in clouds of greater thickness. As evidence for the fall of moderate or heavy rain from non-freezing clouds did not exist at that time, he came to the conclusion that coalescence occurred with small drops but not with larger ones. Houghton(3) made calculations of a similar nature but did not give any evidence for or against the process.

Since that time several accounts(4–6) have appeared of moderate to heavy rain having been seen to fall from clouds which did not reach freezing level,

---

* Division of Radiophysics, C.S.I.R.O., University Grounds, Sydney.

suggesting that the mechanism by which rain forms in these clouds is worthy of further investigation. In the present paper an attempt is made to explain the production of rain from non-freezing clouds on the assumption that after the initial growth of cloud droplets by condensation, a small proportion of their number grows further by coalescence. Calculations of the rate of growth are made, similar to those of Findeisen and Houghton, and these are used to determine the paths taken by individual drops within a cloud. The effect of varying the cloud parameters is investigated and it is found that the final size of the raindrops formed by coalescence depends mainly on the vertical air velocity in the cloud. Their size is less affected by variations in the cloud water content and the degree of supersaturation. It is also deduced that under appropriate conditions a concentration of drops may occur within a relatively narrow range of heights above the cloud base. This effect has been observed in both flight and radar experiments.

II. THEORY

Consider the course of events in a single column in the atmosphere in which the air is ascending with uniform vertical velocity and with no horizontal motion. It is known that the air cools adiabatically as it ascends and that cloud droplets form when it becomes saturated. The droplets then grow by condensation at a rate which is determined mainly by the degree of supersaturation of the air. If the mean upward air velocity is v, the motion of any one cloud droplet is given by

$$\frac{dh}{dt}=v-u, \qquad (1)$$

where h=height above the base of the cloud,

t=time,

u=terminal velocity of the cloud droplet.

Stokes's law will hold for cloud droplets of the size we are considering and their terminal velocity is given by

$$u=\frac{g\rho D^2}{18n}, \qquad (2)$$

where g=acceleration due to gravity,

D=drop diameter,

n=viscosity of air,

ρ=density of water.

The full expression for growth by condensation has been given by Frössling(7) but it is sufficient for the present approximate discussion to use a much simpler one derived from that given by Houghton(8) :

$$D^2=8kQ\,.\,\triangle q.t, \qquad (3)$$

where k=diffusion coefficient of water vapour in air,

Q=saturated vapour density,

$\triangle q$=percentage supersaturation.

Inserting (2) and (3) in equation (1), gives for the motion of the cloud droplet

$$\frac{dh}{dt}=v-\frac{8kg}{18n}\rho\,.\,Q\triangle q.t. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Integrating and inserting the condition $h=0$ where $t=0$, we have

$$h=vt-\frac{2kg}{9n}\rho\,.\,Q\triangle q.t^2, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

which gives the height above the cloud base reached by a cloud droplet after the lapse of time t. For typical values of cloud parameters the expression $\frac{2kg}{9n}\rho\,.\,Q\triangle q.t^2$ is small, indicating that the cloud droplets will lag very little behind the ascending air and continue to rise with it as long as the up current persists.

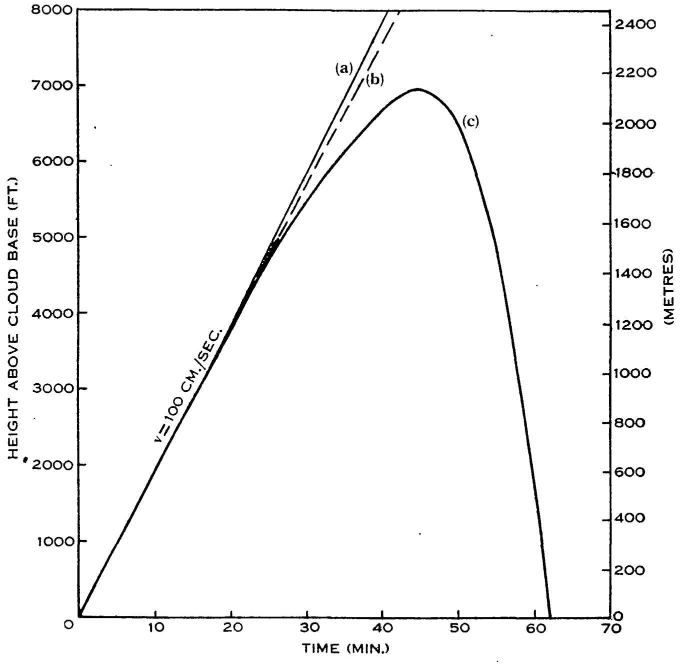


Fig. 1.—The motion of (*a*) the air, (*b*) cloud droplets, and (*c*) droplets which have grown by coalescence, in a cloud in which the mean upward air velocity is 100 cm./sec., average droplet diameter 20μ and cloud water content 1 g./m.^{3}.

Taking as an example a cloud with a mean upward air velocity $v=100$ cm./sec., a degree of supersaturation 0·1 per cent. and the usual value of the other constants at 10 °C., the motion of the cloud droplets with time is found to be as in curve (*b*) of Figure 1, the motion of the air being given by (*a*). It is evident that unless some mechanism of growth comes into action in addition to condensation, none of the droplets will grow large enough to fall against the upward air current in a cloud of reasonable thickness. If, however, as a consequence of the growth by condensation or due to turbulence and mixing, cloud droplets of different diameter appear together in any small sample of cloud, they will fall relative to one another in the gravitational field and collisions will occur. If, on collision, coalescence takes place, rapid growth will follow.

The question whether water droplets coalesce on collision has been debated for many years. Rayleigh(9) is often quoted as showing that coalescence does not take place. His conclusions were drawn from experiments on the behaviour of drops in two impinging jets of tap water, so that they do not necessarily apply to droplets in clouds. He found that his drops did not normally coalesce in dust-free air, but he was at pains to point out that coalescence did take place remarkably easily in the presence of small quantities of contamination in the water, in the presence of many forms of dust, including atmospheric dust, or if there were small differences of potential between the drops. It is clear from his results that surface effects are crucial in determining whether coalescence takes place. In the absence of direct evidence as to whether cloud droplets coalesce on contact or not, it is proposed to consider the consequences which would arise if every collision in a cloud resulted in coalescence, and see whether the results are in agreement with observations.

Langmuir(10) has made a study of the conditions which obtain when a drop of diameter D falls through a cloud of slightly smaller droplets. On the above assumption that each collision results in coalescence, he derived a quantity E, the collection efficiency of the drop, or the fraction of droplets in its path which is picked up by coalescence. If the velocity of fall of the larger drop is u, the cloud has a liquid water content w grams per unit volume, and the rate of fall of the cloud droplets is negligible, then the rate of change of volume of the larger drop is

$$\frac{d.\ \mathrm{vol.}}{dt}=\frac{\pi D^2}{4}\,.\,wuE, \quad \ldots\ldots\ldots\ldots \quad (6)$$

and its rate of change of diameter is

$$\frac{dD}{dt}=\frac{wuE}{2}. \quad \ldots\ldots\ldots\ldots \quad (7)$$

Langmuir showed that E increases from zero, or a very small value for a small drop, to nearly unity for a drop of raindrop diameter falling through a cloud of droplets. Some experimental checks of collection efficiency have been made in this Laboratory giving values in fair agreement with those of Langmuir; the latter have therefore been used in the calculations which follow.

Let us now trace the growth by coalescence of a drop in falling through a cloud which is assumed to have a mean upward air velocity v of 100 cm./sec., a cloud water content w of 1 g./m.^{3} and an average droplet diameter of 20μ after the initial growth period. If, due to turbulence or mixing, two droplets of slightly different diameter come together and coalesce, they will form a drop of approximately twice the mass. This will fall relative to the others. Its subsequent growth has been calculated from equation (7) above, using the values of E given by Langmuir, the terminal velocity u derived from Stokes's law for small droplets and that given by Laws(11) for larger drops. As both the terminal velocity and the collection efficiency change in a complex fashion with drop diameter, the calculation has been made numerically. It is found that growth by coalescence is slow at first and only of the same order of magnitude as the growth by condensation. However, u and E increase rapidly with drop size and some time after collisions commence the drop is large enough to remain in suspension in the up current. On further growth it falls through the rising air, and, growing still further in its downward passage, it finally emerges as a raindrop from the base of the cloud.

The trajectory obtained in this way is given in Figure 1 (*c*), indicating that under the conditions specified the drop comes into equilibrium in the air current after 45 minutes at a height of 6950 ft. above the cloud base. It descends to the base in another 17 minutes, the whole process taking 62 minutes. The corresponding curve of height against diameter (Fig. 2) can then be obtained by a simple step, indicating that the final size of the drop is 1·55 mm.—a raindrop of moderate size.

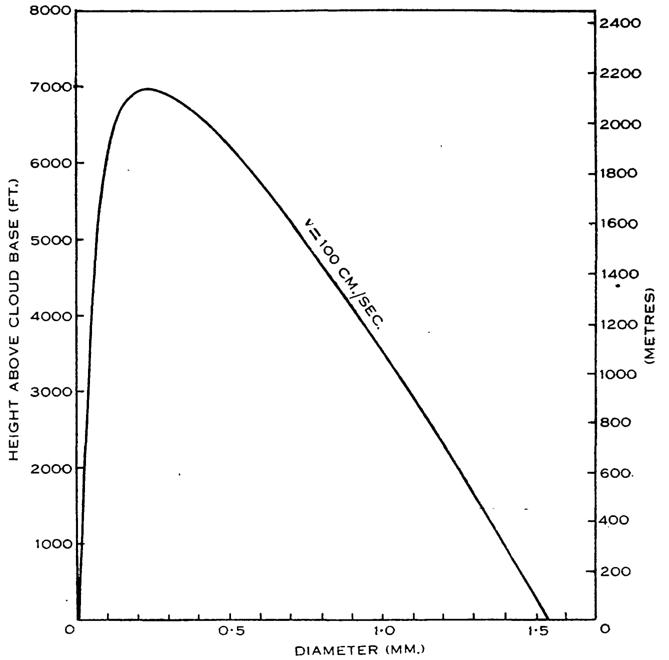


Fig. 2.—The variation of diameter with height of a drop growing by coalescence in a cloud in which the mean upward air velocity is 100 cm./sec., average droplet diameter 20μ and the cloud water content 1 g./m.^{3}.

III. The Effect of Vertical Air Velocity

Some early calculations showed that of the cloud parameters likely to affect the final drop size, vertical air velocity had a predominant effect. The effect of varying the vertical air velocity over a wide range of values has therefore been investigated and a series of curves similar to those of Figures 1 and 2 computed for mean rates of ascent v=10, 25, 50, 100, and 200 cm./sec. If condensation

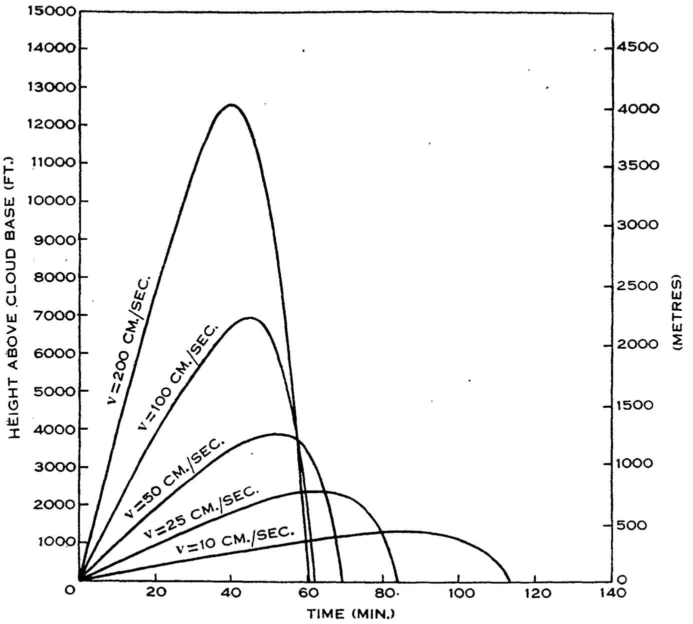


Fig. 3.—The trajectories of drops which grow by coalescence in clouds having a range of vertical air velocities from 10 to 200 cm./sec.

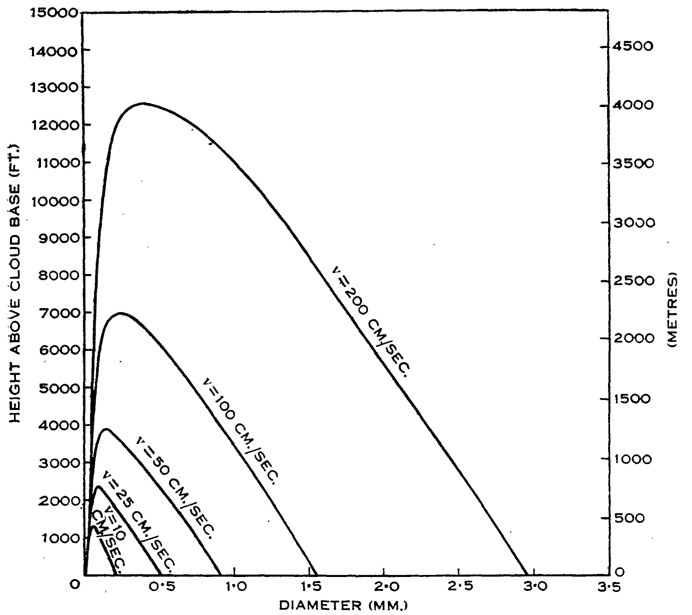


Fig. 4.—The change in drop diameter with height for a range of vertical air velocities.

took place on the same number of nuclei in each case, the degree of supersaturation attained through the bulk of the cloud would be approximately a linear function of the rate of ascent. It does not follow that this is true in natural clouds but it appears from Howell's(12) treatment of growth by condensation that it is in the right sense. As a first approximation, therefore, it is assumed that the percentage supersaturations corresponding to the above vertical air velocities are 0·01, 0·25, 0·5, 0·1, and 0·2 respectively. A series of height-time curves is then obtained as in Figure 3 and a corresponding series of curves of height against diameter as in Figure 4. It is seen that a whole range of raindrop sizes from 0·2 to 3 mm. is possible for upward air currents varying from 10 to 200 cm./sec. The general conclusion can therefore be drawn that a condensation-coalescence process of the type described is capable of accounting for raindrops of a wide range of sizes and could be the mechanism at work when rain is observed from non-freezing clouds.

It can also be seen from Figures 3 and 4 that if rain forms in the manner postulated, the maximum height reached by the drops will increase with the mean upward air velocity while the size of the drops emerging from the base of the cloud will, in turn, increase with the maximum height attained. The time for the whole process, on the other hand, will be an inverse function of the vertical air velocity, clouds with a low value of up current taking a long time to precipitate.

IV. The Effect of Varying Other Cloud Parameters

The other cloud parameters which are likely to affect the trajectories of raindrops produced by this process are the degree of supersaturation, the cloud water content, and the average size of the cloud droplets. The effect of varying these quantities and of departures from Langmuir's values of collection efficiency has been investigated by repeating the calculations over a wide range of values. In each case it is found that, while the form of the trajectory might vary with changes in the parameters, the final drop size is relatively unchanged.

(*a*) *Degree of Supersaturation*

It follows from the treatment in Section II that the degree of supersaturation will have little direct effect on the final size of the drops since condensation contributes so little to their final mass. However, since the degree of supersaturation determines the initial rate of growth, it will influence the point at which growth by coalescence starts and in this way might affect the final drop size. Taking as an example the cloud conditions specified in Section II, it is found that doubling the degree of supersaturation causes the height of the trajectory to be reduced from 6950 ft. to 5100 ft. and the drop size from 1·55 mm. to 1·25 mm., that is, a change of 2 : 1 in percentage supersaturation causes a change of only 20 per cent. in the final drop size. It will be noted that the change is in the inverse sense, an increase in the degree of supersaturation causing a decrease in the height attained and a corresponding decrease in the size of the drop.

(*b*) *Cloud Water Content*

In the same way, the effect of changing the cloud water content has been calculated for a range of values from 0·5 gram per cubic metre to 1·5 grams per cubic metre, the other conditions being unchanged, namely, vertical air velocity $v=100$ cm./sec., a degree of supersaturation of 0·1 per cent. and a mean cloud droplet size of 20μ. The curves of drop size against height shown in Figure 5

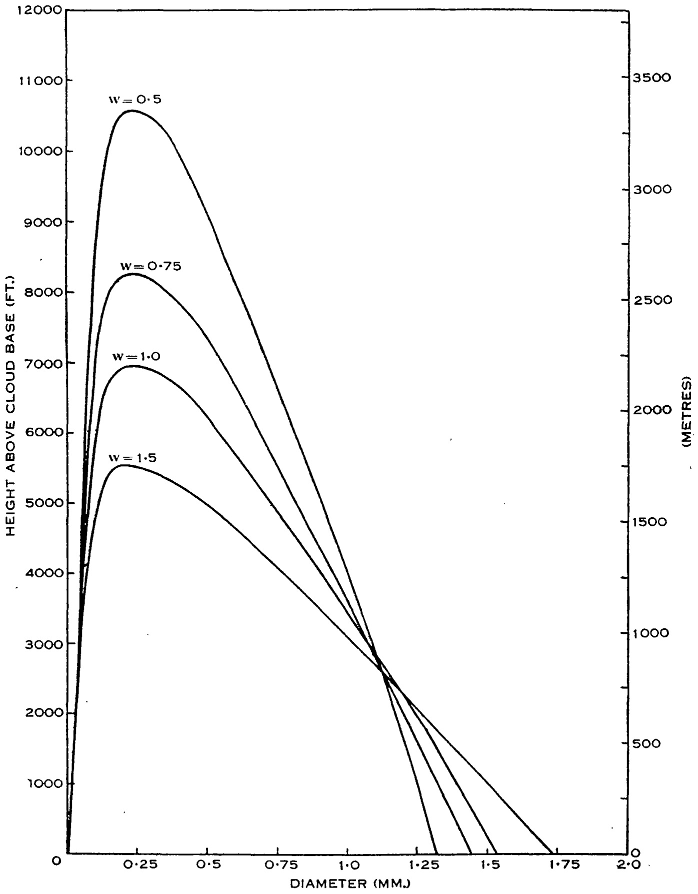


Fig. 5.—The change in drop diameter with height for different values of cloud water content.

are obtained, indicating that an increase in the ratio 3 : 1 in the cloud water content causes an increase of only 33 per cent. in the final drop size. This is due to the fact that, while the rate of growth increases with increased cloud water content, the length of the path over which growth takes place is reduced. It can be seen therefore that the final size of drops growing by coalescence is relatively insensitive to changes in cloud water content.

(*c*) *Collection Efficiency*

In the same way, the effect of departures from Langmuir's figures for collection efficiency has been computed for cloud conditions which are otherwise unchanged. This has been done by assuming that no growth by coalescence occurs until the cloud droplets attain a mean diameter of 20μ, after which growth takes place at fixed values of *E* ranging from 0·25 to 1. The curves of drop size against height obtained in this way are given in Figure 6, showing again that while the maximum height of the trajectory changes, the final size of the raindrops increases by only 30 per cent. for an increase of 4 : 1 in the value of *E*.

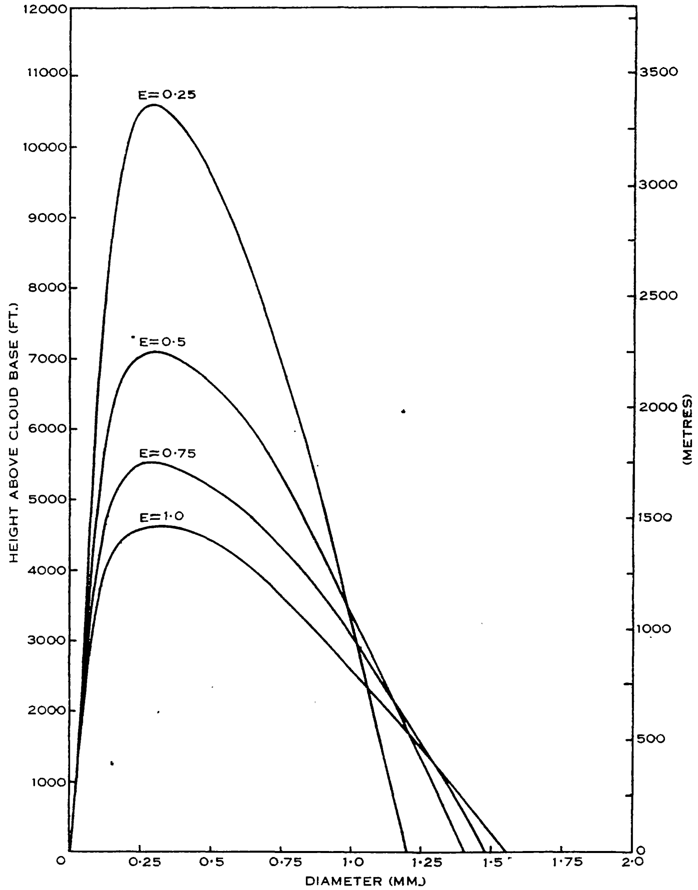


Fig. 6.—The change in drop diameter with height for different fixed values of " collection efficiency ".

(*d*) *Cloud Droplet Size*

Cloud droplet size does not appear in the formula for growth by coalescence, but for a fixed cloud water content the effect of variation in the size of the cloud droplets appears as a change in the collection efficiency *E*. Calculations have been made for droplet diameters of 20, 30, and 40μ, giving the curves of Figure 7, which again show a relatively small change in the final drop size. Calculations for cloud droplets smaller than 20μ are probably no longer valid due to uncertainties in the value of E but it is evident that the collision rate will fall off rapidly, which means that both the rate of growth and the fraction of cloud droplets growing by coalescence are much reduced when the cloud droplets are small. This might well be a critical condition for rain to form by the coalescence process. Knowledge of the collision process is not exact enough for firm conclusions to be drawn, however, but it is probable that the average droplet diameter in a cloud must grow to at least 20μ before coalescence becomes important.

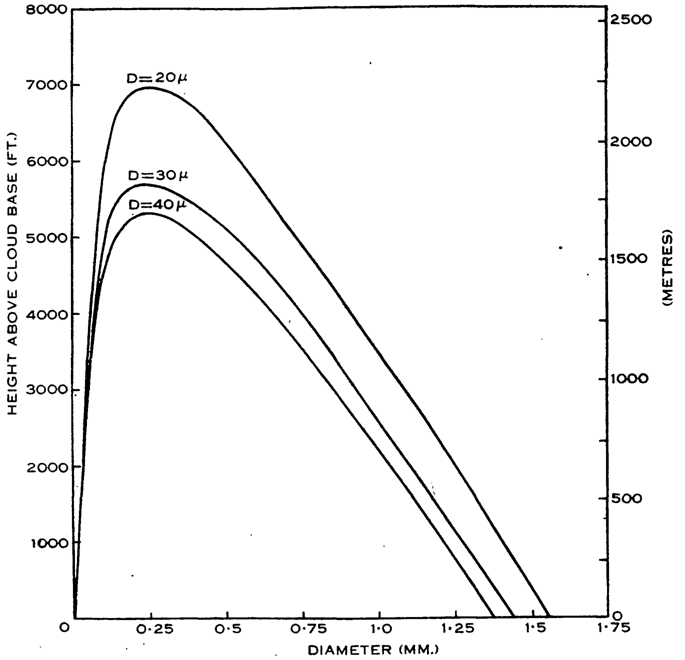


Fig. 7.—The change in drop diameter with height for clouds with an average cloud droplet diameter of 20, 30, and 40μ respectively.

V. Experimental Observations

A complete experimental check of the theory would require measurement of the diameter and number of raindrops, the vertical air velocity and the cloud water content at different levels in typical clouds. Many of these measurements are difficult to carry out and it may be some time before they are accomplished. In the meantime, evidence in support of the theory has been obtained from observations of rain from non-freezing clouds by ground radar equipment and aircraft flights in the vicinity of Sydney, Australia. In general, it is found that such rain takes on two distinct forms, one in which the rain intensity increases gradually downwards through the cloud, the other in which there is a layer type of formation with a concentration of relatively large drops at some height above the cloud base.

The first of these may be regarded as the general case in which droplets start growing by coalescence at various levels in the cloud giving rise to a variety of trajectories of the type shown in Figure 3. It is known that the intensity of radio waves scattered from raindrops within a cloud is proportional to $\Sigma\, ND^6$, where N is the raindrop density and D the diameter of the drops. When a cloud of the above type is observed by radar, therefore, the raindrops would first be detected at that point in their descent where the quantity $\Sigma\, ND^6$ is sufficient to give a measurable signal. Any subsequent increase in diameter of the raindrops as they fell through the cloud would give a further increase in signal intensity.

The second form, that in which a concentration of water drops is found at some height above the cloud base, shows a horizontal band structure when observed by radar. It is thought to be a particular case in which there is some factor at work tending to make all the raindrop trajectories coincide or nearly coincide. It will be shown that in these circumstances there would be a concentration of drops at the top of the trajectory.

A detailed description will now be given of the two types of non-freezing rain which have been observed.

(*a*) *The General Case*

A typical example of the first kind occurred on the morning of December 1, 1949, when light rain fell inland of the Radiophysics Laboratory. Maritime air was moving across the coast from an easterly direction with 6/8 cumulus cloud, the cloud base being at 2500 ft. An aircraft was operating overhead at the time and it was found that the cloud tops were generally at 10,000 ft. where the temperature was $+2\frac{1}{2}$ °C. A few heads appeared some 500 ft. higher but did not attain freezing level at any time.

Rain fell west of the Laboratory for several hours from 0930 hours onwards. It was observed on an SCR 717 radar operating on a wavelength of 10 cm. and modified to scan from horizon to horizon through the zenith. The sensitivity of the set was such that approximately 10^4 raindrops per cubic metre, each $0 \cdot 5$ mm. diameter, would just be detected if they filled the beam at a distance of 10,000 ft. The echo pattern observed at 1002 hours is shown in Plate 1 which is, in effect, a side elevation view in an east-west plane through the rain area. The observing point is at the centre of the baseline and the bright semicircles are range markers at 5000, 10,000, and 15,000 ft. respectively. The rain echoes appear wholly on the right of the picture, the echoes on the left being from objects on the ground east of the observing point. The echo pattern changed only slowly during three hours' observation and was of the same general appearance throughout. In Plate 1 the rain echoes extend downwards from an average height of 6000 to 7000 ft. with the exception of the most distant column which extends from 10,000 ft. The gradation of echo intensity is not well reproduced in the plate, but a measure of echo intensity against height has been obtained by means of a microphotometer scanning vertically along the rain columns at the points marked (*a*), (*b*), and (*c*). The results are plotted in

Figure 8 in arbitrary units of echo intensity against height in feet. It is seen that there is satisfactory correspondence between them, all three showing an increase in echo intensity and therefore of rain intensity downward through the cloud. The rapid decrease of intensity below 2000 ft. is not significant as it is probably due to the shielding of the rain echoes near the ground by objects in the vicinity of the radar set.

The column structure shown in Plate 1 is very characteristic of this type of rain. The separate columns are probably related to individual convective cells in the manner described by Byers and Braham(13) for more active cumulus clouds.

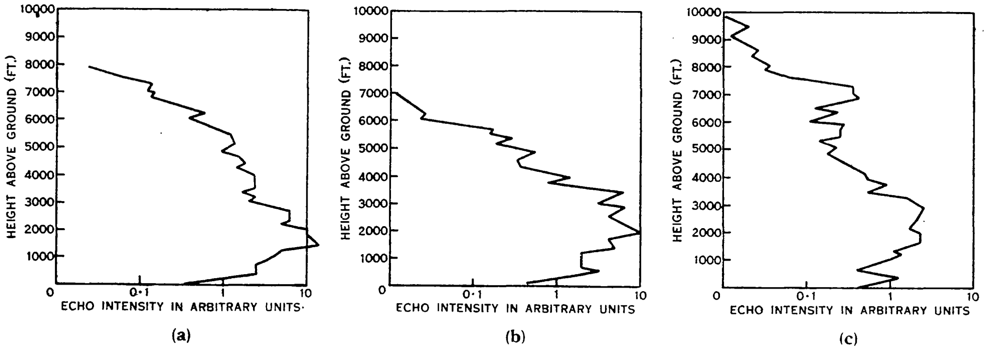


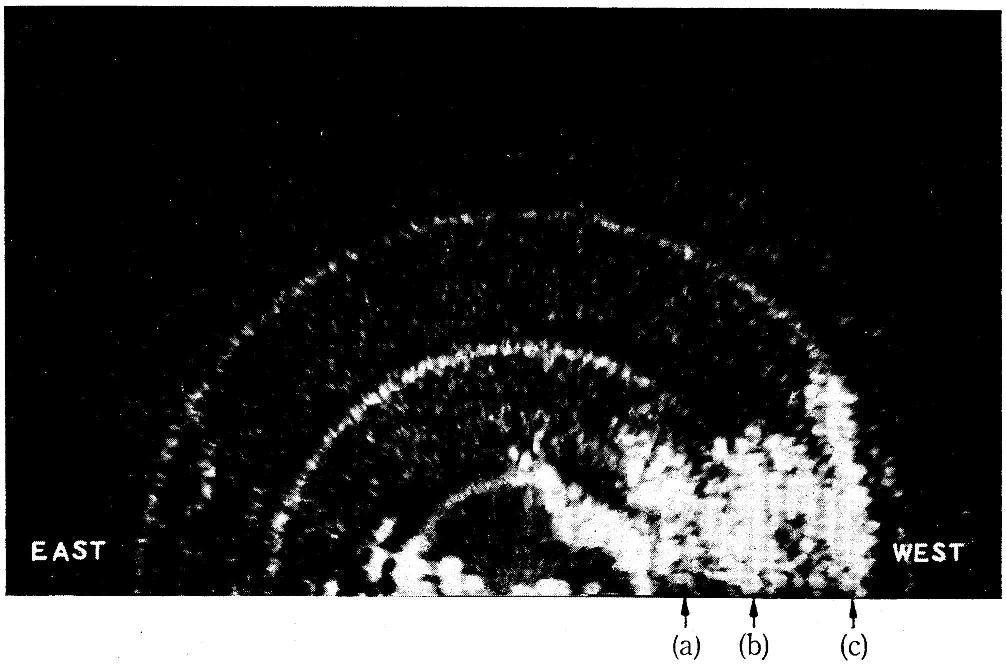
Fig. 8.—The radar echo intensity in arbitrary units plotted against height for three vertical sections (*a*), (*b*), and (*c*) through the rain area shown in Plate 1.

(*b*) *The Horizontal Band Structure*

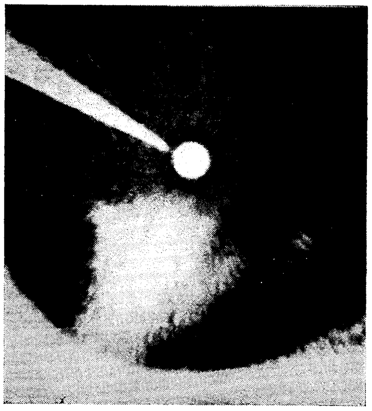
Considering now the particular case in which a concentration of drops appears at some level in the cloud, it is convenient first to estimate how the raindrop density and the rain water content would vary with height in a cloud in which the drop trajectories tend to coincide. Considering only those drops which are moving downward relative to the ground, if n is the number of raindrops of a given size crossing unit area per second, then N, the number of raindrops per unit volume at any level, is given by

$$N=\frac{n}{v-u}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

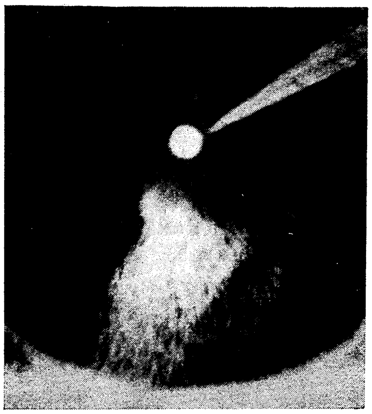
This clearly increases upward from the base of the cloud and tends to infinity at the top of the trajectory. Numerical values of drop density have been calculated for the cloud conditions assumed in Section II, namely an upward air velocity of 100 cm./sec. and a cloud water content of 1 gram per cubic metre, giving the curve shown dotted in Figure 9. In the same way, the rain water content at any level is given by $\frac{\pi N D^3}{6}$, and this has also been computed in terms of n, giving the values shown in the full line in Figure 9. Unlike raindrop density, the rain water content first falls off with increasing height, and then tends to infinity at the top of the trajectory due to the very great increase in raindrop density N.



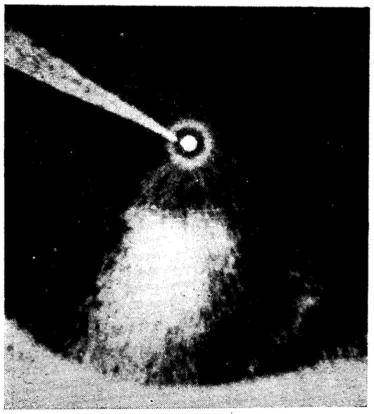
Radar echoes of rain falling from a non-freezing cloud on December 1, 1949.



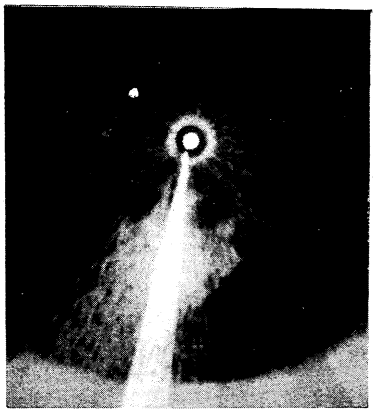
(a) 1427 hours



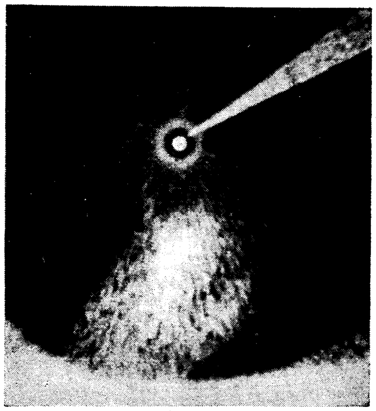
(b) 1446 hours



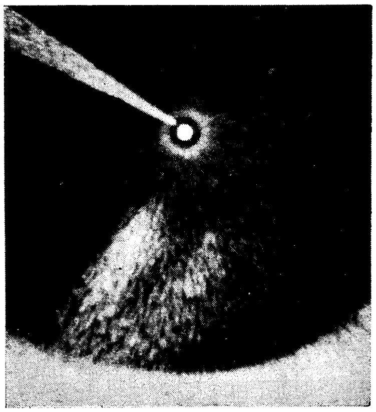
(c) 1450 hours



(d) 1455 hours



(e) 1500 hours



(f) 1504 hours

Airborne radar observations of rain falling from a non-freezing cloud on December 1, 1949. The aircraft height was 14,000 ft. in all cases except (*a*) when it was 12,500 ft.

Because of the assumption that all the drops have identical trajectories, the drops appear to be concentrated in a region of infinitesimal width and the rain water content appears to be infinite. In practice the trajectories are unlikely to coincide exactly and the region would have finite width and finite water content.

It would be expected, therefore, that, if a cloud were producing rain by the condensation-coalescence process and the raindrop trajectories tended to coincide, a concentration of water drops or an increase in rain water content would occur at some height above the cloud base. This, in fact, has been observed by Smith(14) and the author in flights through cumulus clouds which were producing or about to produce rain. Some examples of this effect follow.

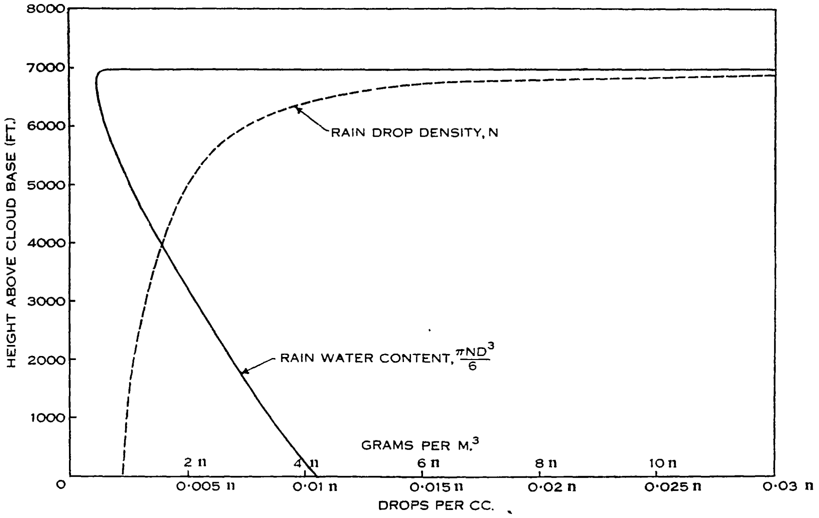


Fig. 9.—The variation with height of raindrop density and rain water content, both in terms of *n* the number of raindrops crossing unit area in unit time. The cloud conditions are as specified in Figure 1.

(i) *Flight Observations of June* 1, 1948.—On one of the occasions described by Smith, namely that of June 1, 1948, he was able to make measurements of raindrop size, drop density, and rain water content at a variety of levels. The instrument used for this purpose was an impactor in which a sensitive paper strip was moved at a uniform speed past a slot exposed to the air stream. The raindrops were recorded as spots on the paper and an estimate of the drop size obtained from the spot size by calibration. The minimum drop size capable of giving a record was approximately 0·1 mm. The raindrop diameters obtained are not claimed to be accurate, but the instrument is thought to give good comparative figures of drop density and water content at the different levels.

The meteorological conditions during the flight have been described in some detail by Smith. Light to moderate rain was falling from a cloud which approached, but did not reach, freezing level. The greatest height reached by the cloud was 8400 ft. where the temperature was $+\frac{1}{2}$ °C. and the base of the cloud was at 1500 ft. Flights were made through it at 8000, 7000, and 6000 ft. and underneath the base at 1300 ft. No ice or snow particles were observed in the cloud and it was found that the rain water content at the top was considerably higher than elsewhere. Measurements made with the impactor gave the results shown in Table 1. They show that the mean drop diameter at the top of the cloud was about 0·3 mm. and was 0·7 mm. just below the cloud base, indicating that the drops grew as they fell through the cloud.

TABLE 1

MEASUREMENTS OF RAINDROP DENSITY AND RAIN WATER CONTENT—JUNE 1, 1948

Height (ft.)	Estimated Diameter (mm.)	<0·2	0·2 to 0·4	0·4 to 0·6	0·6 to 0·8	0·8 to 1·0	1·0 to 1·2	1·2 to 1·4	1·4 to 1·6	Totals
8000	Drop density N/m.^{3}	54,000	9,200	1,300	220	18	—	—	—	64,840
	Water content mg./m.^{3} ..	25	130	85	40	7	—	—	—	287
7000	Drop density N/m.^{3}	14,000	1,100	130	18	—	—	—	—	15,250
	Water content mg./m.^{3} ..	7	15	8	3	—	—	—	—	33
6000	Drop density N/m.^{3}	9,000	400	50	10	—	—	—	—	9,460
	Water content mg./m.^{3} ..	5	6	3	2	—	—	—	—	16
1300	Drop density N/m.^{3}	15,200	1,160	480	220	70	14	5	2	17,150
	Water content mg./m.^{3} ..	7	16	31	40	27	10	6	4	141

The total water content at the different levels is plotted in Figure 10 and if a smooth curve is drawn through them it corresponds closely in form to that given by the theory in Figure 9. The total drop densities have been plotted in the same way, giving the dotted curve of Figure 10. This also agrees with that derived from the theory in showing a maximum in the upper part of the cloud, but it increases again towards the cloud base. It is thought that this is due to the fact that the drops grew in falling through the cloud and a certain number which were too small to be detected by the instrument at the top of the cloud had grown sufficiently for them to be counted at the base. Qualitatively, therefore, the observations agree with the theory in showing an increase in drop diameter as the drops fall through the cloud, and in showing both a maximum drop density and a maximum rain water content near the top of the cloud.

(ii) *Ground Radar Observations.*—If a cloud producing rain in the manner just described were observed on a radar set, the echo intensity along a vertical section, being proportional to ΣND^6, would be approximately of the same form as the rain water content curve of Figure 9. It would therefore be different from that shown in Plate 1, the strongest signal coming not from the falling rain, but from the region where the drops were in suspension.

It is already well known that an intense radar echo is obtained from a horizontal band in many clouds from which rain is falling. It is called the radar " bright band " and has a very characteristic and clearly defined form. In most of the reported instances it has been observed at or just below the freezing level and the explanation(15) which has been advanced for its presence invokes the melting of ice particles or snow-flakes as they fall through freezing level.

Observations made with the radar set referred to in Subsection V (*a*) show that in addition to the bright band at freezing level, other band structures sometimes occur at heights so far removed from freezing level that they are unlikely to be connected with the melting phenomenon. Two cases which were observed in detail occurred on June 1 and June 6, 1949.

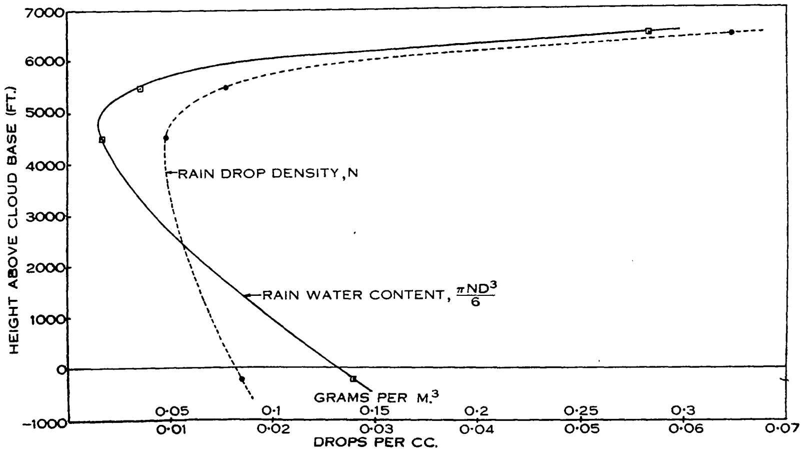


Fig. 10.—Experimental observations of raindrop density and rain water content at different heights in a non-freezing cloud.

June 1, 1949.—On this occasion light showers fell during the morning, the cloud base being at 2000 ft. The cloud tops as determined by aircraft observation were uniform and between 5000 and 6000 ft. When rain was falling, radar echoes were received from a horizontal band overhead at 5000 ft., the band forming and re-forming as the showers passed. It was clearly defined but of low intensity and approximately 1000 ft. thick. A radiosonde record taken at 1800 hours on the same day at Rathmines, some 50 miles north of the point of observation, is given in Figure 12 (*a*), showing that the freezing level was at 7500 ft. and the temperature in the vicinity of the band from +3 to +5 °C. Furthermore, a distinct inversion existed at 5000 ft. which was probably responsible for limiting the clouds to about this level, and for limiting the raindrop trajectories to a particular height.

The average diameter of the raindrops during a typical shower was measured by allowing them to fall on sensitized paper, and found to be 0·5 mm. Reference to Figure 4 indicates that this agrees closely with the value to be expected from the theory for a radar band 3000 ft. above the cloud base.

June 6, 1949.—The conditions were similar to those on June 1 except that drizzle only was falling. The cloud base was at 1500 ft. and the cloud tops estimated to be between 7000 and 8000 ft. A radar band 1000 ft. thick was observed at a height of 3000 ft. fluctuating in intensity with the rainfall. The radiosonde record for that day shown in Figure 12 (*b*) indicates that freezing level was at 6500 ft. and that the temperature in the vicinity of the radar band was +8 °C. Unlike the previous occasion, there was no evidence of an inversion or change of lapse rate at this height. The average drop diameter was 0·35 mm. and the height of the radar band 1500 ft. above the cloud base, again in approximate agreement with the theory.

(iii) *Airborne Radar Observations of December* 1, 1949.—The conditions already described during the morning of December 1, 1949 persisted in the afternoon and a flight was made about fifty miles inland from the coast to observe any further rain which might occur. The flight took place in an aircraft of the Royal Australian Air Force operated by a special detachment of the Aircraft Research and Development Unit based at Richmond, N.S.W. A temperature sounding made during the ascent gave the curve shown in Figure 12 (*c*), the probable error in height being 100 ft. and in temperature 1 °C. The cloud base was at 4000 ft. and the cloud tops generally at 10,500 ft. where the temperature was +4 °C. As in the morning, a few cumulus heads pushed up approximately 1000 ft. higher, but at no time did any clouds in the area reach freezing level, which was at 13,000 ft.

TABLE 2

VARIATION IN HEIGHT OF TOP OF CONVECTIVE CLOUD—DECEMBER 1, 1949

Time (hr.)	Height of Top of Cloud (ft.)	Temperature (°C.)
1400	11,000	+4
1419	11,500	+3
1422	12,000	+2
1450	12,500	+1
1520	10,500	+4

At 1400 hours a small head appeared some 500 ft. above the surroundings and it was selected for special observation. It rose steadily during the next hour to a maximum height of 12,500 ft. as indicated in Table 2, after which it collapsed rapidly to the 10,500 ft. level. Flights were made through it at 1419 and 1421 hours at a height of 11,000 ft., just below the top of the cloud. It would be classed as " very wet " for a cumulus cloud and, as would be expected from the temperature at that level, it contained no ice or snow. The aircraft was fitted with a radar set and during both flights through the cloud a substantial rain echo was observed between the aircraft and the ground. Thereafter flights were made over the top of the cloud at approximately 5 minute intervals, keeping the rain area under observation. The radar set was an SCR 717 similar to that employed for the ground observations, with the antenna mounted so that its axis of rotation was along the fore-and-aft line of the aircraft. As a result the indicator gave a radar cross-section through a plane passing through the aircraft at right angles to the line of flight, instead of the normal plan-position display. Typical photographs of the scan are shown in Plate 2 in which the small bright circle corresponds to the position of the aircraft, the echo pattern immediately below it to radar echoes from the rain, and the bright area below to the echo from the ground. The ground echo appears as an arc of a circle rather than a horizontal line owing to the finite beam width of the antenna system.

A sequence of photographs taken between 1427 and 1504 hours over the region giving the most intense rain echo appears in Plate 2. The lateral extent of the shower was approximately 2 miles in both an east-west and a north-south direction and the photographs show that for the greater part of the time the rain extended from a height of 10,000 ft. to the ground, which was 3000 ft. above sea-level. The echo pattern in the east-west plane showed a distinct shear corresponding to the fact that the winds up to 5000 ft. were generally from the east and above that height from the west. The echo intensity was maintained from the time it was first observed at 1419 until 1450 hours, after which it gradually decreased and disappeared at 1515 hours. The aircraft then descended to the base of the cloud to establish its position and at 1530 hours found only light drizzle between the base of the cloud and the ground. A point of considerable importance in relation to the present theory is that the echo intensity remained strong until 1450 hours, that is, while the cloud was in the process of building up, but fell off in intensity as soon as the cloud had passed its maximum development.

Due to the limited range of echo intensities which can be recorded on a pictorial scan, the photographs in Plate 2 are not suitable for a determination of radar echo intensities. During the flight over the top of the cloud at 1455 hours, therefore, measurements of radar echo intensity against height were made on a separate cathode ray tube which displayed the echo amplitude against height along the line defined by the bright sector in Plate 2 (*d*). These are plotted in Figure 11 as radar echo intensity in decibels above receiver noise level against height in feet, allowance being made for the known variation of echo intensity with distance from the point of observation. Three such measurements were made during a single north-south traverse at 15, 25, and 35 seconds after 1455 hours respectively. All three show a decided maximum in the region between 8500 and 9000 ft. Reference to Figure 12 (*c*) indicates that this corresponds in height to a slight inversion in the temperature lapse rate.

These results are again in qualitative agreement with the theory in showing a region of high water content at a definite height within the cloud.

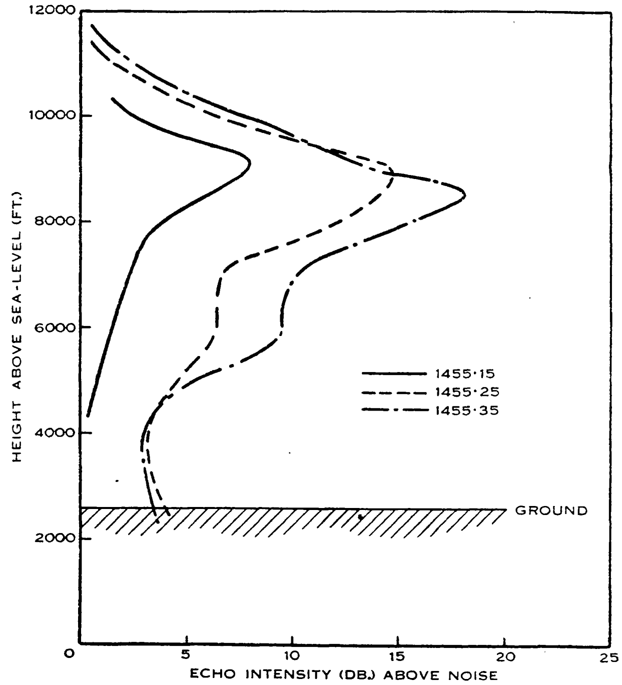


Fig. 11.—Experimental observations of radar echo intensity against height of rain echoes in a non-freezing cloud on December 1, 1949.

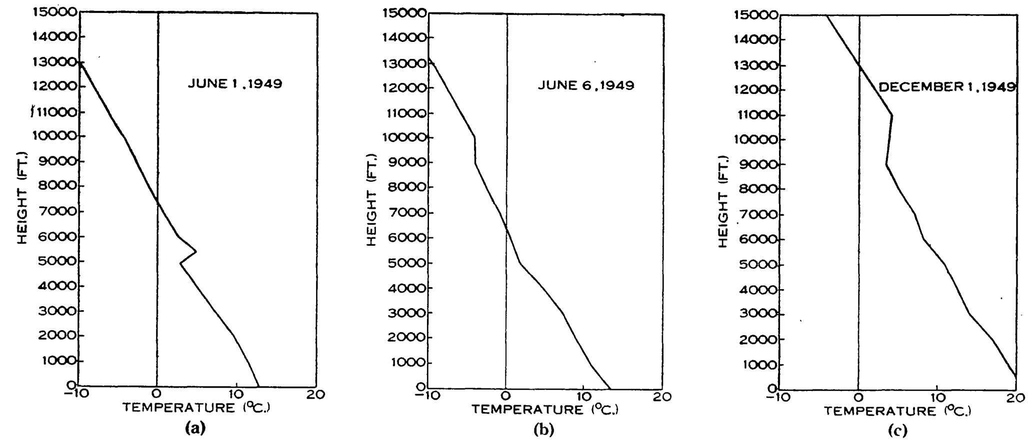


Fig. 12.—The 1800 hour radiosonde records for Rathmines on June 1 and June 6, 1949, and the aircraft sounding for December 1, 1949.

VI. Factors Tending to make Drop Trajectories Coincide

It appears from the foregoing account that the factor most likely to give a high rain water content at a particular height is a decrease or a discontinuity in the vertical air velocity. This may be associated with :

(i) A temperature inversion or a change of lapse rate in the atmosphere.

(ii) The top of a convective cell.

The example of December 1, 1949, was of the first kind, while that of June 1, 1948, was of the second kind. On June 1, 1949, the concentration of water appeared both at the top of the cloud and at an inversion, while on June 6, 1949, it appeared to be near the middle of the cloud but there was no evidence for a change of lapse rate at that level. The radiosonde record, however, referred to a different place and a different time and it is quite possible that a discontinuity in the lapse rate was present either locally or of a magnitude too small to be measured.

It might be suggested that the results obtained on June 1, 1948, and June 1, 1949, when the concentration of water drops was observed at the top of the cloud, could be accounted for by the mechanism of radiation cooling postulated by Reynolds(16). On both of these occasions, however, the cloud tops were exposed to the sun and were probably experiencing a net gain of heat by radiation rather than a loss. It is, therefore, unlikely to have been the mechanism at work and, as it could not explain those cases in which the concentration of water drops occurred near the centre of the clouds, the author is led to the conclusion that coalescence was the predominant mechanism in the examples cited in the present paper.

VII. Conclusion

Calculations have been made of the results to be expected if cloud droplets coalesce when they come into collision in natural clouds. The mechanism postulated is one in which the cloud droplets first grow by condensation in their ascent through the cloud, some fraction of their number then growing further by coalescence. Those which grow by coalescence ascend until they can no longer be sustained in the upward air current, after which they fall back through the cloud. They grow still further in their descent to the cloud base and finally emerge as rain. It is found that the mechanism is capable of accounting for raindrops of a wide range of sizes and might therefore be the process at work when rain has been observed to fall from clouds consisting wholly of water droplets.

The maximum height attained by the drops and the final diameter of the raindrops emerging from the base of the cloud are shown to be nearly a linear function of the vertical air velocity. The largest raindrops would therefore be expected to fall from clouds with the greatest convective activity. The time for rain to form by the process should, on the other hand, be inversely proportional to the vertical air velocity, clouds of low activity taking a long time to precipitate. These results are consistent with the properties of natural rain.

Experimental observations of rain falling from non-freezing clouds have distinguished two main types. The first of these appears to be the general case in which, due to turbulence or lack of uniformity within the cloud, the drops have a variety of trajectories. It is characterized by a gradual increase of raindrop diameter or rainfall intensity downward through the cloud. The second corresponds to the case in which the raindrop trajectories tend to coincide, in which case a concentration of relatively large drops would be expected to form at some height above the cloud base. This has been observed experimentally as a region from which intense radar echoes are received and as a region of high water content during aircraft flights through rain-producing clouds. The many points of agreement between these observations and the deductions from the theory lead to the conclusion that coalescence plays an important part in the formation of rain from non-freezing clouds.

Since the great majority of clouds do not produce rain, it is appropriate to conclude by considering briefly the conditions which need to be met before rain can form by the condensation-coalescence process. Apart from the basic requirement that coalescence shall occur as a result of collisions between cloud droplets, the following five conditions are necessary:

(1) A distribution of cloud droplet sizes must exist so that the droplets have an opportunity of falling relative to one another and coming into collision. It is probable that this condition is, in fact, met in the great majority of clouds.

(2) The width of the cloud must be such that drops which grow by coalescence will not be carried out of the cloud by a wind shear or another similar factor. This requirement is common to almost any theory of rain formation.

Coming now to the conditions which arise from the theory as developed in the present paper:

(3) The cloud droplets must attain a certain minimum size before collisions are frequent enough to give a reasonable number of raindrops. The present state of knowledge on the collision process is not good enough to define this limit at all accurately.

(4) The vertical air current in a cloud must be maintained long enough for the growth process to be completed. It is a matter of observation that many convective clouds do not meet this requirement, going through their whole cycle of growth and dissipation in a shorter time than that required for raindrops to form by coalescence.

(5) For a given upward air velocity the depth of the cloud must be greater than that required for the drops which grow by coalescence to come into equilibrium in the upward air current.

If any of these conditions are not met, then rain is unlikely to form by the condensation-coalescence process.

VIII. Acknowledgments

The author is grateful to Mr. E. J. Smith, Division of Radiophysics, C.S.I.R.O., for permission to include his experimental observations of drop density and rain water content, and to Mr. J. Warner for assistance with the radar observations. Grateful acknowledgment is also made to the pilots and crew of the special detachment of A.R.D.U. which made the flight observations possible.

IX. References

(1) Bergeron, T.—*P.V. Mét. Un. Géod. Géophys. Int.* Lisbon. pp. 156-78 (1933).
(2) Findeisen, W.—*Met. Z.* **56** : 365-8 (1939).
(3) Houghton, H. G.—*Bull. Amer. Met. Soc.* **19** : 152-9 (1938).
(4) Wexler, H.—*Bull. Amer. Met. Soc.* **26** : 156 (1945).
(5) Kotsch, W. J.—*Bull. Amer. Met. Soc.* **28** : 87-9 (1947).
(6) Hunt, T. L.—*Met. Mag.* **78** : 26 (1949).
(7) Frössling, N.—Gerlands *Beitr. Geophys.* **52** : 170-216 (1938).
(8) Houghton, H. G.—*Physics* **4** : 419 (1933).
(9) Rayleigh, Lord.—*Sci. Pap.* **1** : 372-6 ; **2** : 103-15. (Cambridge Univ. Press, 1900.)
(10) Langmuir, I.—*J. Met.* **5** : 175-92 (1948).
(11) Laws, J. O.—*Trans. Amer. Geophys. Un.* **22** : 709-21 (1941).
(12) Howell, W. E.—*J. Met.* **6** : 134-49 (1949).
(13) Byers, H. R., and Braham, R. R.—*J. Met.* **5** : 71-86 (1948).
(14) Smith, E. J.—*Quart. J.R. Met. Soc.* (Communicated.)
(15) Byers, H. R., and Coons, R. D.—*J. Met.* **4** : 75-81 (1947).
(16) Reynolds, O.—*Collective Works* **1** : 214. (Cambridge Univ. Press, 1900.)