Cloud microphysics

Claudia Emde

Meteorological Institute, LMU, Munich, Germany

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Overview of cloud physics lecture

- Atmospheric thermodynamics
  - gas laws, hydrostatic equation
  - 1st law of thermodynamics
  - moisture parameters
  - adiabatic / pseudoadiabatic processes
  - stability criteria / cloud formation
- Microphysics of warm clouds
  - nucleation of water vapor by condensation
  - growth of cloud droplets in warm clouds (condensation, fall speed of droplets, collection, coalescence)
  - formation of rain
- Microphysics of cold clouds
  - homogeneous nucleation
  - heterogeneous nucleation
  - contact nucleation
  - crystal growth (from water phase, riming, aggregation)
  - formation of precipitation
- Observation of cloud microphysical properties
- Parameterization of clouds in climate and NWP models

Summary L1 - The ideal gas equation

- Equation of state: relation between p, V, T of a material
- Equation of state for gases ⇒ ideal gas equation

$$pV = mRT \qquad p = \rho RT \qquad p\alpha = RT$$

- R - gas constant for 1 kg of gas
- $\alpha = 1/\rho$ - specific volume of gas (V occupied by 1 kg of gas at specific p and T)
- Boyle's law (T=const.) and Charles' laws (p=const., V=const.)

Sir Robort Boyle (1627–1691)

Jacques Charles (1746–1823)

Images from Wikipedia

Summary L1 - Definitions

- gram-molecular weight (mole), e.g. 1 mol H_2O = 18.015 g
- number of moles $n = m/M$
- number of molecules in 1 mole N_A=6.022·10^{23} (Avogadro's number)
- Avogadro's hypothesis: gases containing the same number of molecules occupy the same volume
- universal gas constant R^*=8.3145$JK^{-1}mol^{-1}$ $\Rightarrow pV = nR^*T$
- Boltzmann's constant $k=R^*/N_A$

Amedeo Avogadro (1776–1856)

Ludwig Boltzmann (1844–1906)

Images from Wikipedia

Mixture of gases

- Dalton's law: total pressure exerted by a mixture of gases is equal to sum of partial pressures ($p = p_d + e$)
- dry air (mixture of atmospheric gases excluding water vapor):
  - $p_d \alpha_d = R_d T$
  - p_d "partial pressure" of dry air
  - apparent molecular weight
    $M_d = \frac{\sum_i m_i}{\sum_i m_i / M_i} = 28.97 \frac{\mathrm{g}}{\mathrm{mol}} \Rightarrow R_d = 1000 \frac{R^*}{M_d} = 287.0 \frac{\mathrm{J}}{\mathrm{Kkg}}$
- water vapor
  - $e \alpha_v = R_v T$
  - e - partial pressure of water vapor
  - $M_w = 18.0167 \frac{\mathrm{g}}{\mathrm{mol}} \Rightarrow R_v = 1000 \frac{R^*}{M_w} = 461.51 \frac{\mathrm{J}}{\mathrm{Kkg}}$
- $\frac{R_d}{R_v} = \frac{M_w}{M_d} \equiv \epsilon = 0.622$
- virtual temperature: $p = \rho R_d T_v$ with $T_v \equiv \frac{T}{1 - \frac{e}{p}(1 - \epsilon)}$

The hydrostatic equation

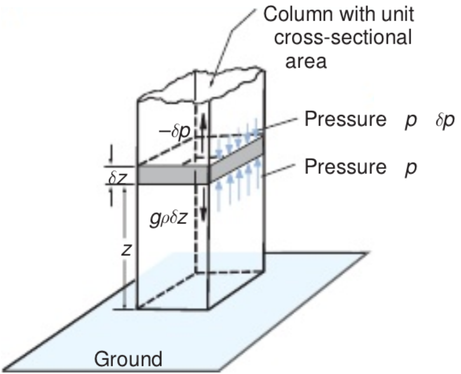


Figure from Wallace and Hobbs

$$\frac{\partial p}{\partial z} = -g\rho \qquad g dz = -\alpha dp$$

Sir Issac Newton (1642–1727)
Image from Wikipedia

First law of thermodynamics

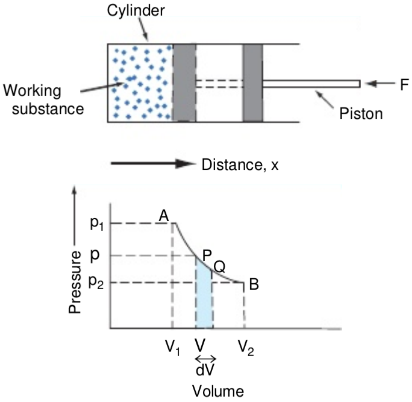


Fig. 3.4 Representation of the state of a working substance in a cylinder on a p–V diagram. The work done by the working substance in passing from P to Q is $p\,dV$, which is equal to the blue-shaded area. [Reprinted from *Atmospheric Science: An Introductory Survey*, 1st Edition, J. M. Wallace and P. V. Hobbs, p. 62, Copyright 1977, with permission from Elsevier.]

Figure from Wallace and Hobbs

energy conservation

$$
\begin{aligned}
dq &= du + dw \\
dq &= du + p d\alpha \\
dq &= c_p dT - \alpha dp \\
\ldots
\end{aligned}
$$

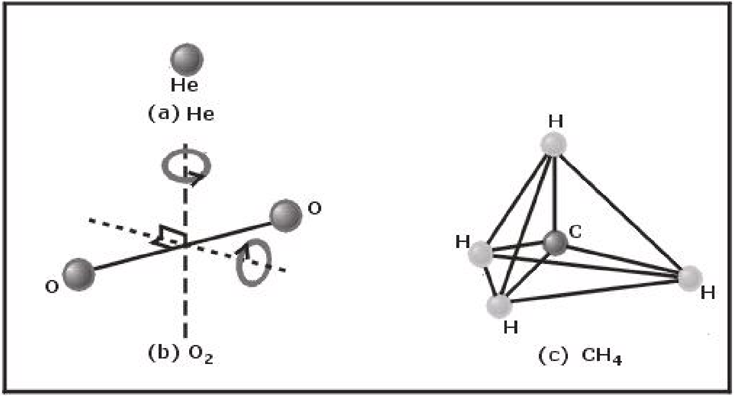
Specific heats

- specific heat at const. V: $c_V = \left(\frac{dq}{dT}\right)_{V=const} = \left(\frac{du}{dT}\right)_{V=const}$
- specific heat at const. p: $c_p = \left(\frac{dq}{dT}\right)_{p=const}$

$$c_p = c_V + R$$
$$c_V = f\frac{R}{2}$$

f-degrees of freedom

- for dry air $\Rightarrow f = 7$ (O_2 and N_2: 2-atomic linear molecules)
  $c_V = 717\frac{\mathrm{J}}{\mathrm{K}}$, $c_p = 1004\frac{\mathrm{J}}{\mathrm{K}}$



http://www.tutorvista.com

Enthalpy

Assume that heat is added to system so that α increases, $V = const$.

$$\Delta Q = (u_2 - u_1) + p(\alpha_2 - \alpha_1) \quad = (u_2 + p\alpha_2) - (u_1 + p\alpha_1)$$
$$= h_2 - h_1$$

Enthalpy of unit mass of a material:

$$h \equiv u + p\alpha$$
$$\ldots$$
$$dh = c_p dT \Rightarrow h = c_p T \qquad (\text{with} \quad h = 0 \quad \text{at} \quad T = 0)$$

h corresponds to the heat required to raise the temperature of a material from 0 to T at p=const.

Application to atmospheric layer

Assumptions:

- layer is at rest and in hydrostatic balance
- layer is heated by radiative transfer (p of overlying air const.)

$\Rightarrow$ air within layer expands and does work on overlying air by lifting it against gravitational force ($dq = c_p dT - \alpha dp$).
This can be written in terms of enthalpy h and geopotential ϕ:

$$dq = d(h + \phi) = d(c_p T + \phi)$$

Definition of geopotential: $d\phi \equiv gdz = -\alpha dp$ (work that must be done against Earth's gravitational field to raise mass of 1 kg from sea level to that point)

$$\phi(z) = \int_0^z gdz$$

Adiabatic processes

adiabatic = change in physical state without heat exchange $\Rightarrow dq = 0$

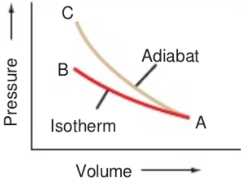


Fig. 3.5 An isotherm and an adiabat on a p–V diagram.

Figure from Wallace and Hobbs

$$dq = du + p d\alpha$$

T rises in adiabatic compression

T=const. in isothermal process

$$T_C > T_B \Rightarrow p_C > p_B$$

Concept of air parcel

Assumptions:

- *molecular mixing can be neglected* (in Earth's atmosphere only important above ≈105 km and for 1 cm layer above surface), i.e. mixing can be regarded as exchange of macroscale "air parcels"
- *parcel is thermally insulated from it's environment*, i.e. T changes adiabatically as parcel rises or sinks, p always adapts to environmental air, which is assumed to be in hydrostatic equilibrium
- *parcel moves slow enough*, i.e. the macroscopic kinetic energy is a negligible fraction of the total energy

Dry adiabatic lapse rate

for adiabatic processes:

$$d(c_p T + \phi) = 0 \Rightarrow -\frac{dT}{dz}_{\text{dry parcel}} = \frac{g}{c_p} \equiv \Gamma_d$$

Γ_d – dry adiabatic lapse rate (change of T with z)

Example for Earth's atmosphere:

- g=9.81 $\frac{m}{s^2}$, c_p=1004 $\frac{J}{K}$ $\Rightarrow \Gamma_d$=9.8 $\frac{K}{km}$
- Actual lapse rate (for moist air) is smaller than Γ_d.

Potential temperature

The potential temperature θ is the temperature that the air parcel would have if it were expanded or compressed adiabatically to standard pressure p_0 (generally p_0=1000 hPa)

Poisson's equation

$$\theta = T \left(\frac{p_0}{p} \right)^{R/c_p}$$

θ is conserved during adiabatic transformations $\Rightarrow$ very useful parameter in atmospheric thermodynamics (most processes adiabatic)

Moisture parameters

- mixing ratio: $w = \frac{m_v}{m_d}$
  typically a few g/kg in mid-latitudes to 20 g/kg in tropics
- specific humidity: $q = \frac{m_v}{m_v+m_d} = \frac{w}{w+1}$
  w≈0.01 $\rightarrow q \approx w$
- virtual temperature for given mixing ratio: $T_v \approx T(1 + 0.61w)$
  for T=30°C and w=20g/kg ⇒ T_v-T=3.7°C

Saturation vapor pressures

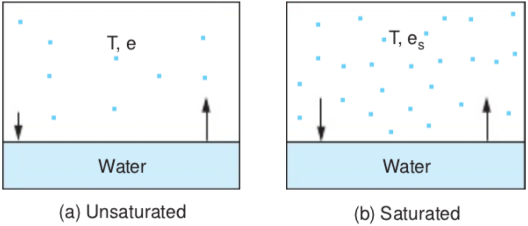


Fig. 3.8 A box (a) unsaturated and (b) saturated with respect to a plane surface of pure water at temperature *T*. Dots represent water molecules. Lengths of the arrows represent the relative rates of evaporation and condensation. The saturated (i.e., equilibrium) vapor pressure over a plane surface of pure water at temperature *T* is e_s as indicated in (b).

Figure from Wallace and Hobbs

- equivalent definitions for water and ice

Saturation vapor pressure

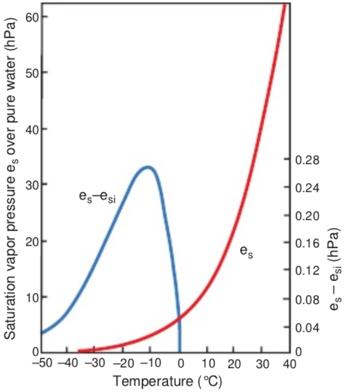


Fig. 3.9 Variations with temperature of the saturation (i.e., equilibrium) vapor pressure e_s over a plane surface of pure water (red line, scale at left) and the difference between e_s and the saturation vapor pressure over a plane surface of ice e_{si} (blue line, scale at right).

Figure from Wallace and Hobbs

- evaporation rate from ice less than from water : $e_s(T) > e_{si}(T)$

  $\Rightarrow$ ice particle in water-saturated air grows due to deposition of water vapor on it (important for formation of precipitation)

Moisture parameters ctd.

- Saturation mixing ratio w_s:
  $w_s = \frac{m_{vs}}{m_d} = \cdots = \epsilon \frac{e_s}{p - e_s} \approx 0.622 \frac{e_s}{p}$ (since for atmospheric T: $p \gg e_s$)
- Relative humidity RH:
  $RH = 100 \frac{w}{w_s} = 100 \frac{e}{e_s}$ [%]
- Dew point T_D:
  temperature to which air must be cooled at p=const., so that air becomes saturated w.r.t. water (equivalent def. for frost point)

  measurement of T_D yields $RH = \frac{e_s(T_D,p)}{e_s(T,p)}$

Lifting condensation level (LCL)

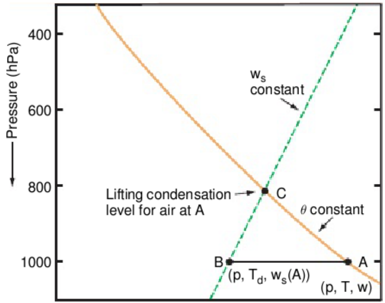
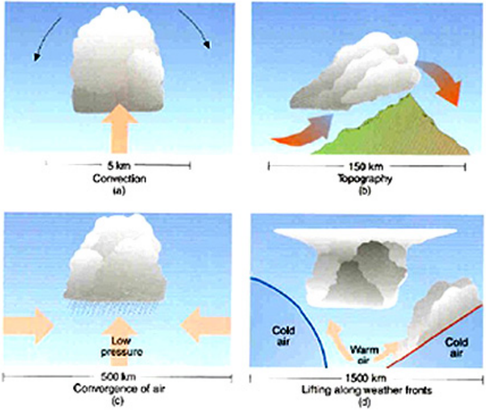


Fig. 3.10 The lifting condensation level of a parcel of air at A, with pressure p, temperature T, and dew point T_d, is at C on the skew $T - \ln p$ chart.

Figure from Wallace and Hobbs

- LCL: level to which moist air parcel can be lifted adiabatically before it becomes saturated w.r.t. water
- during lift: w=const., θ=const., w_s decreases until $w_s = w$ at LCL

Lifting condensation level (LCL)



from /rst.gsfc.nasa.gov

Lifting condensation level (LCL)

from Wikipedia

Latent heats

- If heat is added to system ⇒ change in T *or* change in phase
- phase transition: Δu completely used for changes in molecular configuration in presence of intermolecular forces

- Latent heat of melting L_m: heat that is required to convert unit mass of a material from solid to liquid phase without change in T, equal to latent heat of freezing
- melting point: T at which phase transition occurs
- for water at 1013hPa, 0°C ⇒ $L_m = 3.34 \cdot 10^5 \frac{J}{kg}$

- latent heat of vaporization or evaporation L_v defined equivalently
- for water 1013hPa, 100°C (boiling point) ⇒ $L_v = 2.25 \cdot 10^6 \frac{J}{kg}$

Saturated adiabatic and pseudoadiabatic processes

- air parcel rises ⇒ T decreases with z until saturation is reached
- further lifting ⇒ condensation of liquid water (or deposition on ice) ⇒ release of latent heat ⇒ rate of decrease in T reduced

Saturated adiabatic process

All condensation products remain in parcel, process still adiabatic and reversible

Pseudoadiabatic process

Condensation products fall out, process is irreversible. Not adiabatic since products carry out **small** amount of heat.

Saturated adiabatic lapse rate

$$\Gamma_s = -\frac{dT}{dz} \approx \frac{\Gamma_d}{1 + \frac{L_v}{c_p}\left(\frac{dw}{dT}\right)_p}$$

- Γ_s varies with p, T; in contrast to Γ
- since condensation releases heat: $\Gamma_s < \Gamma$
- typical values:
  4 K/km near ground in warm humid airmasses
  6-7 K/km in middle troposphere
  near tropopause, Γ_s only slightly smaller than Γ (e_s very small, no condensation)

Equivalent potential temperature θ_e

θ_e is the potential temperature θ of the air parcel when all water vapor has condensed out so that it's saturation mixing ratio is zero.

$$\theta_e \approx \theta \exp\left(\frac{L_v w_s}{c_p T}\right)$$

(During "Föhn", T and θ increase, RH decreases, θ_e remains constant)

Static stability for unsaturated air

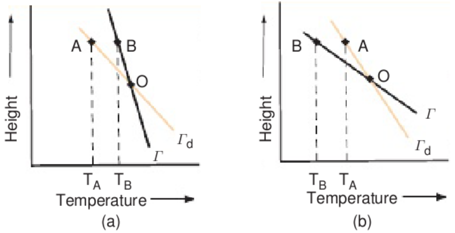


Fig. 3.12 Conditions for (a) positive static stability ($\Gamma < \Gamma_d$) and (b) negative static instability ($\Gamma > \Gamma_d$) for the displacement of unsaturated air parcels.

Figure from Wallace and Hobbs

- atmospheric layer with actual lapse rate Γ less than dry adiabatic lapse rate Γ_D $\Rightarrow$ stable stratification, positive static stability
- $\Gamma > \Gamma_D \Rightarrow$ unstable stratification, positive static stability (not persistant in free atmosphere due to strong vertical mixing)
- $\Gamma > \Gamma_D \Rightarrow$ neutral

Gravity waves

For stably stratified layers, so called gravity waves may form.

buoyancy oscillation of air parcel

$$z'(t) = z'(0) \cos Nt$$

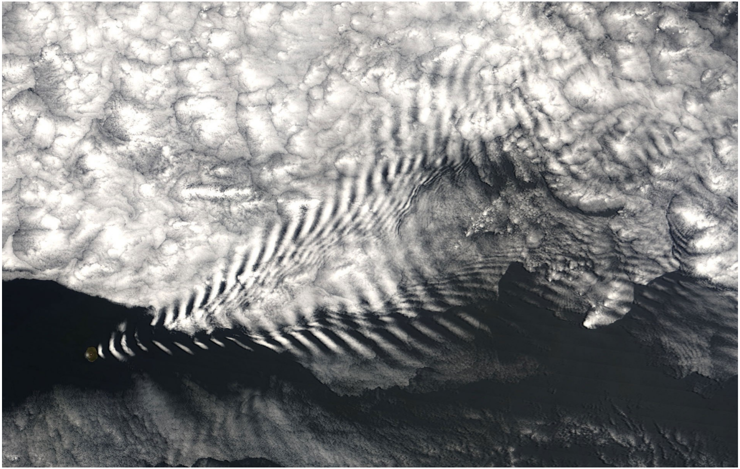
Brunt-Väisälä frequency

$$N = \left(\frac{g}{T} (\Gamma_d - \Gamma) \right)^{1/2}$$

Gravity waves

from Wikipedia

Gravity waves



from Wikipedia

Gravity waves

from www.weathervortex.com

Inversions

from Wikipedia

Inversions

Photo by B. Mayer, taken at Heimgarten

Static stability for saturated air

- if air parcel is saturated $\Rightarrow$ T decreases with height at Γ_S
- with same arguments as for unsaturated air parcel
  - $\Gamma < \Gamma_S$ – stable
  - $\Gamma = \Gamma_S$ – neutral
  - $\Gamma > \Gamma_S$ – unstable

Conditional and convective stability

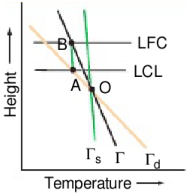


Fig. 3.16 Conditions for conditional instability ($\Gamma_s < \Gamma < \Gamma_d$). Γ_s and Γ_d are the saturated and dry adiabatic lapse rates, and Γ is the lapse rate of temperature of the ambient air. LCL and LFC denote the *lifting condensation level* and the *level of free convection*, respectively.

Figure from Wallace and Hobbs

- atmospheric layer with actual lapse rate between Γ_S and Γ_D $\Rightarrow$ conditional instability
- Level of free convection (LFC) $\Rightarrow$ from this level parcel is unstable, is carried upward in absence of forced lifting
- vigorous convective overturning can occur if vertical motions are large enough to lift air parcel beyond LFC

Convective overshooting

from Wikipedia